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The influence of the drag force due to the interstitial gas on granular flows down a chute

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Abstract

Fully-developed steady flow of granular material down an inclined chute has been a subject of much research interest, but the effect of the interstitial gas has usually been ignored. In this paper, new expressions for the drag force and energy dissipation caused by the interstitial gas (ignoring the turbulent fluctuations of the gas phase) are derived and used to modify the governing equations derived from the kinetic theory approach for granular–gas mixture flows, where particles are relatively massive so that velocity fluctuations are caused by collisions rather than the gas flow. This new model is applied to fully-developed, steady mixture flows down an inclined chute and the results are compared with other simulations. Our results show that the effect of the interstitial gas plays a significant role in modifying the characteristics of fully developed flow. Although the effect of the interstitial gas is less pronounced for large particles than small ones, the flowfields with large particles are still very different from granular flows which do not incorporate any interactions with the interstitial gas. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In the past two decades, many competing theories have been proposed for modelling rapid granular flows (see, e.g., Ogawa et al., 1980; Jenkins and Savage, 1983; Lun et al.,

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1984; Jenkins and Richman, 1985; Abu-Zaid and Ahamadi, 1990). The numerical results using a kinetic theory approach for cases where the dissipation is relatively small, and the particle concentrations range from small to moderate, have shown quite good agreement with experimental data. However, the effect of the interstitial gas has always been neglected because the interaction between the particles and the gas has been regarded as too complex to consider.

Recently, Shen and Ackermann (1982) and Shen et al. (1988) began to tackle this problem by simply adding an energy dissipation term in the equation for the balance of energy. Ma and Ahmadi (1988) derived a constitutive theory for stresses and fluctuational energy flux which included the effect of interstitial fluid from the Boltzmann's equation. Aragon (1995) introduced a turbulent stress and a shear stress into the kinetic-momentum equation. Moreover, he treated the interstitial fluid as an energy dissipater in the energy equation. A more sophisticated way to integrate the effect of interstitial fluid is described by Jenkins and McTigue (1995), who derived the constitutive relations by considering the lubrication forces between neighbouring spheres for a slow flow of concentrated suspensions. Similarly, Sangani et al. (1996) studied suspensions where the mean relative velocity between the particles and the suspension is zero and proposed a convincing expression for the energy dissipation rate derived from the lubrication forces (which may be interpreted as the dissipation caused by interaction between fluctuations in the two phases). However, theoretical modelling of the granular–fluid mixture flow is still at an early stage. The object of this present work is, therefore, to integrate the effect of the interstitial gas into the existing kinetic theory for the rapid granular flow in a more comprehensive and consistent way than before.

Fully-developed, steady chute flows have also been investigated exhaustively in recent years (see, e.g., Ahn et al., 1992; Cao et al., 1996; Johnson et al., 1990; Abu-Zaid and Ahmadi, 1993), even though these studies omitted the effect of interstitial air. Simulation and modelling results are, however, still not in good agreement with experiment, mainly because of difficulties in formulating appropriate boundary conditions and in the rotation of the frictional particles (which is usually ignored). Here, we will use a new model to investigate the granular–air mixture flow down a chute, where both particles and chute wall are non-frictional in order to avoid the errors from omitting the rotational motion of particles. Therefore, the effects due to the interstitial gas will be easily distinguished. The focus of this paper is the difference between dry granular and granular–air mixture flow down a chute in fully developed conditions.

The boundary conditions at the wall are very complicated in a real chute flow and affect the flow profiles substantially. They depend not only on the geometry of the wall but also the physical properties of the wall and particles. So far, this analysis is at an early stage. Slight differences among boundary conditions lead to large disagreements in the solutions derived from competing theories. Here, we will adopt simple boundary conditions, modified from Anderson and Jackson (1992) for a non-frictional wall, which depend on only one unmeasurable parameter. Because our aim is to examine the effects of the interstitial gas, we will use the same boundary conditions in all cases.

Because of the paucity of available experimental data, we will compare our results with simulations of other authors in order to capture and exemplify the differences in flow profiles due to incorporating interstitial gas effects in granular flow calculations.

2. The governing equations

2.1. The governing equations for dry granular flows

The governing equations for rapid flow of dry granular materials are as follows (Lun et al., 1984)

Continuity equation

$$\frac{d\rho}{dt} + \rho\nabla\cdot c_0 = 0, \tag{1}$$

Momentum equation:

$$\rho\frac{dc_0}{dt} + \nabla\cdot p = \rho F, \tag{2}$$

Energy equation:

$$\frac{3}{2}\rho\frac{dT}{dt} + p{:}\nabla c_0 = -\nabla\cdot q - I, \tag{3}$$

where ρ is the bulk density; c_0, the mean bulk velocity; p, the stress tensor; q, the flux vector of pseudo-thermal energy; F, the specific gravity force; I, the collisional dissipation rate of pseudo-thermal energy; T, the 'granular temperature' $= 1/3\langle C^2\rangle$, and C is the fluctuation velocity. The definition of granular temperature is analogous to the definition of a gas temperature. But in the kinetic theory, the gas temperature is defined for a gas in a uniform steady state at rest or in uniform translation. For a gas not in a uniform steady state, the temperature at any point is defined as that for which the same gas, when in a uniform steady state at the same density, would have the same mean thermal energy per molecule at that point (Chapman and Cowling, 1970). In the definition of granular temperature, the granular flow can be at any state (see Ogawa, 1978; Jenkins and Savage, 1983). Therefore, when borrowing results from the kinetic theory of gases, it is important to distinguish this difference.

2.2. The contribution of drag force to the governing equations

We will consider the influence of interstitial gas, usually air, in the above equations (1)–(3). The governing equations are based on an element volume which contains a large number of particles, so that the particles can be regarded as a continuum phase. Because the molecules of gas are much smaller than the granular particles, the microstructure scales of the two phases are very different. The element volume for the particle phase may be too large to allow the stresses acting on it from the interstitial gas to be regarded as a point-tensor in the stress field of the whole flow field. Hence, the effect of the interstitial gas on this element volume should be derived by summing the forces on all the individual particles inside it, based on a velocity distribution function.

If c_r is the instantaneous relative velocity between a particle and the gas, the drag force for one spherical particle, per unit area of the particle body projected on to a plane normal to c_r, is $-(1/2)C_D\varepsilon(1-\varepsilon)\rho_G|c_r|c_r$; hence, the drag force, F_d, per unit volume for one spherical particle may be expressed as (Gidaspow, 1994),

$$F_{\mathrm{d}} = -\frac{3}{4} C_{\mathrm{D}} \frac{\varepsilon(1-\varepsilon)}{d} \rho_{\mathrm{G}} |c_{\mathrm{r}}| c_{\mathrm{r}}. \tag{4}$$

Here, C_{D} is the coefficient of drag force, which may be obtained from experiment, with suitable modification for the high volume fraction of particles; ε is the volume fraction of the gas phase in the element (the void fraction); d is the diameter of the spherical particle (we will only consider uniform spherical particles) and ρ_{G} is the density of the gas. Following Clift et al. (1978), we know that C_{D} is a function of the Reynolds number, Re. The equation for drag coefficient for a single particle, proposed by Kürten et al. (1966), which is valid in the range of Re between 0.1 and 4000, with a deviation of less than 7% from experimental data, is:

$$C_{\mathrm{D}} = \left(0.28 + \frac{6}{\sqrt{Re}} + \frac{21}{Re}\right). \tag{5}$$

However, due to the effect of the other particles in the gas, the equation for the drag coefficient for a single particle in a gas should be corrected. The correction function proposed by Gidaspow and Ettehadieh (1983) is suitable, which gives the coefficient of drag force as

$$C_{\mathrm{D}} = \left(0.28 + \frac{6}{\sqrt{Re}} + \frac{21}{Re}\right) \cdot \varepsilon^{-2.65}, \tag{6}$$

where

$$Re = \frac{\varepsilon \rho_{\mathrm{G}} c_{\mathrm{r}} d}{\mu_{\mathrm{G}}} = \frac{\varepsilon \rho_{\mathrm{G}} (c - v) d}{\mu_{\mathrm{G}}} \approx \frac{\varepsilon \rho_{\mathrm{G}} (c_0 - v) d}{\mu_{\mathrm{G}}}. \tag{7}$$

Here, μ_{G} is the viscosity of the gas; v is the velocity of the gas, which we assume is uniform in the volume element under consideration; and c is the instantaneous velocity of the particles. Although the expression for the coefficient of drag involves the instantaneous velocity of the particles, we will use the average value of Re in the volume element instead of individual values to simplify the modelling and to provide a first order approximation for the effective drag coefficient.

The total force $F_{\mathrm{d(total)}}$ acting on the element volume is then,

$$F_{\mathrm{d(total)}} = \int F_{\mathrm{d}} f \, \mathrm{d}c = -\frac{3}{4} C_{\mathrm{D}} \frac{\varepsilon(1-\varepsilon)}{d} \rho_{\mathrm{G}} \int |c - v|(c - v) f \, \mathrm{d}c. \tag{8}$$

If the particles are relatively massive and nearly elastic, the velocity distribution, f, is close to Maxwellian. Therefore, we obtain,

$$F_{\mathrm{d(total)}} = -\frac{3}{4} C_{\mathrm{D}} \frac{\varepsilon(1-\varepsilon)}{d} \rho_{\mathrm{G}} |c_0 - v|(c_0 - v). \tag{9}$$

The rate of energy dissipation, W, by the drag force may be divided into two parts: W_1, due to the particles' fluctuation velocity and W_2, caused by the difference in the mean velocities of the two phases. Although this is another approximation, it is believed that it is sufficient to capture the essential physics of the dissipation of energy.

Physically, the first part of the rate of energy dissipation, W_1, corresponds to a flow where the mean velocities of gas and particles are equal. Assuming the velocity distribution function is Maxwellian, i.e.

$$f^{(0)} = \frac{n}{(2\pi T)^{3/2}} \exp\left(-\frac{C^2}{2T}\right), \tag{10}$$

then,

$$W_1 = \int F_\mathrm{d} \cdot cf \,\mathrm{d}c \approx -\frac{3}{4} C_\mathrm{D} \frac{\varepsilon(1-\varepsilon)}{d} \rho_\mathrm{G} \int C\vec{C} \cdot \vec{c} f^{(0)} \,\mathrm{d}c, = -\frac{3}{4} C_D \frac{\varepsilon(1-\varepsilon)}{d} \rho_\mathrm{G} \frac{8\sqrt{2}}{\sqrt{\pi}} T^{3/2}. \tag{11}$$

The second part of the rate of energy dissipation, W_2, is due to any difference in the mean velocities, and is given by:

$$W_2 = F_{\mathrm{d(total)}} \cdot (c_0 - v) = -\frac{3}{4} C_\mathrm{D} \frac{\varepsilon(1-\varepsilon)}{d} \rho_\mathrm{G} |c_0 - v| (c_0 - v)^2. \tag{12}$$

Therefore, combined,

$$W = W_1 + W_2 = -\frac{3}{4} C_\mathrm{D} \frac{\varepsilon(1-\varepsilon)}{d} \rho_\mathrm{G} \left(\frac{8\sqrt{2}}{\sqrt{\pi}} T^{3/2} + |c_0 - v| (c_0 - v)^2 \right). \tag{13}$$

Now, we can integrate the drag force acting on the element given by Eq. (9) and the energy dissipation caused by the drag force given in Eq. (13) into the governing equation (1)–(3) for granular–gas mixture flow. Because the density of the gas is negligible compared to that of the particles, the buoyancy force need not be considered. Also, the turbulent fluctuation of the gas phase is regarded as negligible, and added mass, the Basset history term and any aerodynamic lift forces are also ignored. The energy dissipated by the interaction between the fluctuations of the two phases is ignored here, which has been shown to be a major energy dissipation mechanism in dense suspensions (Sangani et al., 1996; Jenkins and McTigue, 1995). For an unbounded rapid flow, e.g., the granular-air flow down a chute considered here, this energy dissipation mechanism is likely to be much less important, however its relative magnitude compared with the other dissipation mechanism will be examined below.

The modified governing equations for momentum and energy therefore become, respectively:

$$\rho \frac{\mathrm{d}c_0}{\mathrm{d}t} + \nabla \cdot p = \rho F - \frac{3}{4} C_\mathrm{D} \frac{\varepsilon(1-\varepsilon)}{d} \rho_\mathrm{G} |c_0 - v| (c_0 - v), \tag{14}$$

$$\frac{3}{2} \rho \frac{\mathrm{d}T}{\mathrm{d}t} + p{:}\nabla c_0 = -\nabla \cdot q - I - \frac{3}{4} C_\mathrm{D} \frac{\varepsilon(1-\varepsilon)}{d} \rho_\mathrm{G} \left(\frac{8\sqrt{2}}{\sqrt{\pi}} T^{3/2} + |c_0 - v| (c_0 - v)^2 \right). \tag{15}$$

3. Constitutive theory

According to Lun and Savage (1987), rotational energy comprises no more than 25% of the translational energy for an infinitely large frictional coefficient. Furthermore, Cao et al. (1996) have shown that the ratio of the rotational fluctuation kinetic energy to the translation fluctuation energy is of the order of μ^2, where μ is the coefficient of friction. Therefore, most theories which include the frictional force omit the effects of this rotating motion. Unfortunately, these theories also depend on many difficult-to-measure parameters, and their application regimes are limited. In order to avoid errors from ignoring the rotating motion and the arbitrary value of some parameters, we restrict ourselves to flows of smooth particles down a smooth, bumpy and slightly inelastic wall. This will allow the influence of the interstitial gas to be clearly distinguished. Moreover, this model is consistent with the assumption of neglecting the aerodynamic lift force caused by rotation of particles in the gas.

For slightly inelastic grains, the stress tensor is composed of the collisional stress, p_c, and the translational stress, p_k. Here, we adopt the expressions for p, q and I reported by Lun et al. (1984), viz.,

$$p = p_k + p_c = \left\{\left[1 + 4\eta(1-\varepsilon)\chi\right]\rho T - \eta\mu_b\nabla\cdot c_0\right\}U - \left\{\frac{2\mu_a}{\eta(2-\eta)\chi}\left[1+\frac{8}{5}\eta(3\eta-2)(1-\varepsilon)\eta\right] + \frac{6}{5}\mu_b\eta\right\}S, \tag{16}$$

where $\eta = (1+e)/2$, where e is the coefficient of restitution between particles; U is the unit tensor and S is the rate-of-shear tensor, given by:

$$S = \frac{1}{2}(c_{0i,\,j} + c_{0j,\,i}) - \frac{1}{3}c_{0k,\,k}\delta_{i,\,j}. \tag{17}$$

Also,

$$\mu_a = \frac{5d\rho_0\sqrt{\pi T}}{96}, \tag{18}$$

and

$$\mu_b = \frac{256\mu_a(1-\varepsilon)^2\chi}{5\pi}. \tag{19}$$

The heat flux is also given by,

$$q = q_k + q_c = -\frac{\lambda_b}{\chi}\left\{\left[1+\frac{12}{5}\eta(1-\varepsilon)\chi\right]\left[1+\frac{12}{5}\eta^2(4\eta-3)(1-\varepsilon)\chi\right] + \frac{64}{25\pi}(41-33\eta)(1-\varepsilon)^2(\eta\chi)^2\right\}\nabla T - \frac{\lambda_b}{\chi}\left[1+\frac{12}{5}\eta(1-\varepsilon)\chi\right]\frac{12}{5}\eta(2\eta-1)(\eta-1)\frac{\mathrm{d}}{\mathrm{d}\varepsilon}\left[(1-\varepsilon)\chi\right]^2\frac{T}{(1-\varepsilon)}\nabla\varepsilon, \tag{20}$$

where,

$$\lambda_b = \frac{25 d\rho_0\sqrt{\pi T}}{16\eta(41-33\eta)}, \tag{21}$$

and

$$I = \frac{48}{\sqrt{\pi}}\eta(1-\eta)\frac{\rho_0(1-\varepsilon)^2}{d}\chi T^{3/2}. \tag{22}$$

In these equations, χ is the radial distribution function. In the present work, we adopt the expression used by Lun and Savage (1987), viz.

$$\chi = \left[1-\frac{1-\varepsilon}{\varepsilon_{\rm m}}\right]^{-2.5\varepsilon_{\rm m}}, \tag{23}$$

where $\varepsilon_{\rm m}$ represents the maximum possible particle fraction of the system.

4. Steady, fully-developed chute flow

We will apply our modified governing equations to steady, fully-developed flow down a smooth, bumpy, inclined chute. We use the flow depth, H, which is the distance from the wall to the free surface, as a control parameter instead of the flow rate because it simplifies the numerical calculations. A Cartesian frame is adopted with y perpendicular to the flow direction. For steady, fully-developed flow, the mean flow velocity, solid volume fraction and granular temperature only vary in the y direction. The inclined angle of the chute is denoted by ξ.

Non-dimensional variables are introduced as follows,

$$\hat{y} = \frac{y}{d}, \quad \hat{u} = \frac{u}{\sqrt{dg}}, \quad \hat{T} = \frac{T}{dg}, \quad \hat{q} = \frac{q}{\rho_0(dg)^{3/2}}, \quad \hat{p}_{yy} = \frac{p_{yy}}{\rho_0 dg}, \quad \hat{p}_{xy} = \frac{p_{xy}}{\rho_0 dg},$$
$$\hat{u}_w = \frac{u_w}{\sqrt{dg}}, \quad \hat{v} = \frac{v}{\sqrt{dg}}, \tag{24}$$

where $\rho = \rho_0(1-\varepsilon)$, and ρ_0 is density of the particle, u is the bulk velocity in the x direction, v is the mean velocity of the gas in an element, p_{xy} is the shear stress and p_{yy} is the normal stress.

The continuity equation is thereby automatically satisfied and, substituting Eqs. (16)–(24) into Eqs. (14) and (15), the governing equations for a steady, fully developed chute flow are then:

(a) momentum equations
x direction

$$\frac{d\hat{p}_{xy}}{d\hat{y}} - (1-\varepsilon)\sin\xi + \frac{3}{4}C_D\varepsilon(1-\varepsilon)\frac{\rho_G}{\rho_0}(\hat{u}-\hat{v})^2 = 0, \tag{25}$$

where, u and v are the mean velocities of the particles and interstitial gas, respectively.

y direction

$$\frac{d\hat{p}_{yy}}{d\hat{y}} + (1-\varepsilon)\cos\xi = 0. \tag{26}$$

(b) energy equation

$$\hat{p}_{xy}\frac{\partial\hat{u}}{\partial\hat{y}} + \frac{\partial\hat{q}}{\partial\hat{y}} = \frac{48}{\sqrt{\pi}}\eta(1-\eta)(1-\varepsilon)^2\chi\hat{T}^{3/2} + \frac{3}{4}C_D\varepsilon(1-\varepsilon)\frac{\rho_G}{\rho_0}\left(\frac{8\sqrt{2}}{\sqrt{\pi}}\hat{T}^{3/2} + (\hat{u}-\hat{v})^3\right). \tag{27}$$

Here, we have assumed the direction of the gas velocity is the same as that of the particle velocity. The following equations are derived from Eqs. (16) and (20),

$$\hat{p}_{xy} = -f_1(\varepsilon)\frac{5}{96}\sqrt{\pi\hat{T}}\frac{d\hat{u}}{d\hat{y}}, \tag{28}$$

$$\hat{p}_{yy} = [1+4\eta(1-\varepsilon)\chi](1-\varepsilon)\hat{T}, \tag{29}$$

$$\hat{q} = -f_2(\varepsilon)\sqrt{\hat{T}}\left[f_3(\varepsilon)\frac{\partial\hat{T}}{\partial\hat{y}} + f_4(\varepsilon)\hat{T}\frac{d}{d\varepsilon}\left[(1-\varepsilon)^2\chi\right]\frac{\partial\varepsilon}{\partial\hat{y}}\right]. \tag{30}$$

where,

$$f_1(\varepsilon) = \frac{1}{\eta(2-\eta)\chi}\left[1+\frac{8}{5}\eta(1-\varepsilon)\chi\right]\left[1+\frac{8}{5}\eta(1-\varepsilon)\chi(3\eta-2)\right] + \frac{768}{25\pi}\eta(1-\varepsilon)^2\chi, \tag{31}$$

$$f_2(\varepsilon) = \frac{25}{16}\frac{\sqrt{\pi}}{\eta(41-33\eta)\chi}, \tag{32}$$

$$f_3(\varepsilon) = \left[1+\frac{12}{5}\eta(1-\varepsilon)\chi\right]\left[1+\frac{12}{5}\eta^2(4\eta-3)(1-\varepsilon)\chi\right] + \frac{64}{25\pi}(41-33\eta)(1-\varepsilon)^2(\eta\chi)^2, \tag{33}$$

$$f_4(\varepsilon) = \frac{12}{5}\eta(2\eta-1)(\eta-1)\left[1+\frac{12}{5}\eta(1-\varepsilon)\chi\right]. \tag{34}$$

The last term in Eq. (30), which has negligible effect on the numerical results according to Johnson et al. (1990), is neglected in the numerical calculation.

5. Boundary conditions

5.1. Boundary conditions at a smooth, bumpy and inclined wall

The boundary conditions have to be prescribed accurately in order to solve Eqs. (25)–(27) for chute flow. But it is difficult to find a mature existing model even for this simple flow. The granular flow at the wall cannot be mediated by the external wall in the same way as classical fluid mechanics. The wall is an integral part of the whole flow field. For rapid granular flows, Hui et al. (1984) proposed boundary conditions by considering the collisions between the grains and the wall. Richman (1988) derived boundary conditions for a bumpy inelastic wall by assuming the velocity distribution function is Maxwellian. Cao et al. (1996) modified this model by integrating the friction effects of wall and particles. Anderson and Jackson (1992) established boundary conditions for rapid chute flow based on the work of Johnson et al. (1990). For the chute flow modelled here, we will adopt their expressions for shear stress and energy dissipation due to the collision between the wall and particles.

The shear stress generated on the thin flow layer above the wall is given as:

$$S = \omega\phi\rho_0(1-\varepsilon)u_{\text{slip}}\sqrt{3T}\chi, \tag{35}$$

where ϕ is a 'specularity factor', which measures the fraction of the momentum of the incident particle transferred to the wall. It has the value of zero for perfectly specular rebound and unity for diffuse scattering. The value of ϕ depends on the coefficient of restitution between particle and wall, as well as the geometry of the wall. The slip velocity of the particles at the wall is denoted by u_{slip}.

The energy dissipation term is also given by:

$$D = \frac{\alpha}{2}\rho(3T)^{3/2}(1-e_{\text{w}})\chi, \tag{36}$$

where, e_{w} is the coefficient of restitution between particle and wall, and α and ω are dimensionless proportionality constants of order unity. Here, we follow Johnson et al. (1990) in adopting:

$$\alpha = \omega = \frac{\pi}{6\varepsilon_{\text{m}}}. \tag{37}$$

From momentum and energy balance at the boundary, we get

$$p_{xy} = S, \tag{38}$$

$$S \cdot u_{\text{slip}} - D = q. \tag{39}$$

Substituting Eqs. (29), (30), (35) and (36) into Eqs. (38) and (39), we obtain the following expressions:

$$\left[1 + 4\eta(1-\varepsilon)\chi\right]\hat{T} = \omega\phi\hat{u}_{\text{slip}}\sqrt{3\hat{T}}\chi, \tag{40}$$

$$\omega\phi(1-\varepsilon)\hat{u}_{\text{slip}}^2\sqrt{3\hat{T}}\chi-\frac{\alpha}{2}(1-\varepsilon)\left(3\hat{T}\right)^{3/2}(1-e_{\text{w}})\chi$$
$$=-f_2(\varepsilon)\sqrt{\hat{T}}\left[f_3(\varepsilon)\frac{\partial\hat{T}}{\partial\hat{y}}+f_4(\varepsilon)\hat{T}\frac{\mathrm{d}}{\mathrm{d}\varepsilon}\left[(1-\varepsilon)^2\chi\right]\frac{\partial\varepsilon}{\partial\hat{y}}\right]. \tag{41}$$

5.2. Boundary conditions at the free surface

At the free surface, $y = H$, the following physical boundary conditions must be satisfied:

$$\varepsilon = 1,\quad \frac{\partial\hat{T}}{\partial\hat{y}} = 0,\quad \frac{\partial\hat{u}}{\partial\hat{y}} = 0. \tag{42}$$

6. The numerical method

The finite difference global scheme described by Reese et al. (1995) for rarefied gas flows is used here. The variables $\hat{u}_i, \hat{T}_i$, and ε_i, $i = 1,2,\ldots,n$, may be treated as a vector $X = (X_i)^{\mathrm{T}}$, whose elements are three-component vectors $X_i = (\hat{u}_i,\ \hat{T}_i,\ \varepsilon_i)^{\mathrm{T}}$. On substituting second order finite difference expressions for the derivatives, Eqs. (25)–(27) with the boundary conditions (40)–(42) become a non-linear system:

$$F(X) = 0. \tag{43}$$

From experimental observations and other computational simulations, large velocity and solid volume fraction gradients occur at the boundaries. The problem is regarded as stiff and the numerical grid should be chosen with care. Here, we adopt the grid step, $\hat{y}$, suggested by Cao et al. (1996):

$$\hat{y}_i = \frac{1}{2}\hat{H}\left[1-\cos\left(\frac{i-1}{n-1}\pi\right)\right],\quad i = 1,2,3,\ldots,n. \tag{44}$$

150 grid points and central finite differences for the first and second derivatives at every point have been used to solve the equations. A global Newton iteration procedure is used to solve the set of nonlinear equations (43). If the variables change is less than 10^{-4} of their absolute values between two consecutive iterations and $F(X_i)$ is also less than 10^{-4} at every point, the iterations are regarded as converged. In the calculation, it was found that good initial guesses are very important in ensuring quick convergence.

7. Results and discussion

The profiles of particle velocity, volume fraction and the granular temperature of dry granular and grain–air mixture flows down a smooth inelastic chute have been calculated, as well as the stresses, energy flux and energy dissipation profiles. In addition, the influence of different particle sizes and different velocities of interstitial air has been studied. The lack of suitable experimental data for comparison means that our results can, at present, be compared only with other simulations of somewhat-artificial cases. However, this does allow some conclusions to be drawn regarding the quantitative and qualitative differences our model introduces when compared with other models which do not include interstitial gas effects.

The parameters of the boundary wall and particles for all cases examined are given in Table 1. These were chosen to enable direct comparison with other reported results.

In Fig. 1, the results predicted by the present model for dry granular flow of non-frictional spheres are shown to be in good agreement with the simulation results of Oyediran et al. (1994) and Cao et al. (1996), both of whom used boundary conditions derived from Richman (1988). This might imply that the specularity factor can express the different boundary geometry conditions for a smooth bumpy wall. As far as the influence of the interstitial gas is concerned (here, the interstitial air is assumed to be at rest), the dimensionless velocity and the square root of the temperature are small when the flow height is the same as the dry flow. This may be understood as the drag force damping both the mean and fluctuation velocities of the particles. But the solid volume fraction does not increase, as might be expected under the condition of constant mass flow rate. Note that, throughout, the flow height is maintained constant, not the mass flow rate. Hence, if the flow rate is constant, the flow heights for the mixture flow and dry flow are different when the flows reach the fully-developed steady state. Using mass flow rate as a design parameter does, however, cause certain difficulties in the numerical solution.

We can see from Fig. 1(a) and (b) that the flow rate of the mixture flow is smaller than that of dry flow if the flow heights are the same. Fig. 1(b) shows that the volume fraction of the particles is nearly uniform from the boundary wall to mid-flow, then decreases rapidly to a very small value. There is a large region near the free surface where the particle volume fraction is nearly zero. For the granular–air mixture flow, Fig. 1(c) shows an interesting feature: the granular temperature decreases from the wall until mid-flow height, then it starts to increase again. Surprisingly, this does not appear in a dry flow. In the following figures, we can observe a similar phenomenon in mixture flows with different particle sizes, velocities of the interstitial air and flow heights. This could be due to the energy being dissipated largely by drag in the air instead of inelastic collisions when the granular temperature is small.

The effect of different sizes of the particles in the granular–air mixture flows can be seen in Fig. 2. For smaller particles, the effect of the interstitial air is larger, as relatively light particles

Table 1
Physical parameters used in the chute-flow calculations

e_w	0.95	ρ_G	1.2 (kg/m^3)
e	0.8	ρ_0	2900 (kg/m^3)
ξ	20.7°	ϕ	0.4
μ_G	1.85×10^{-5} (N s/m^2)	ε_m	0.644

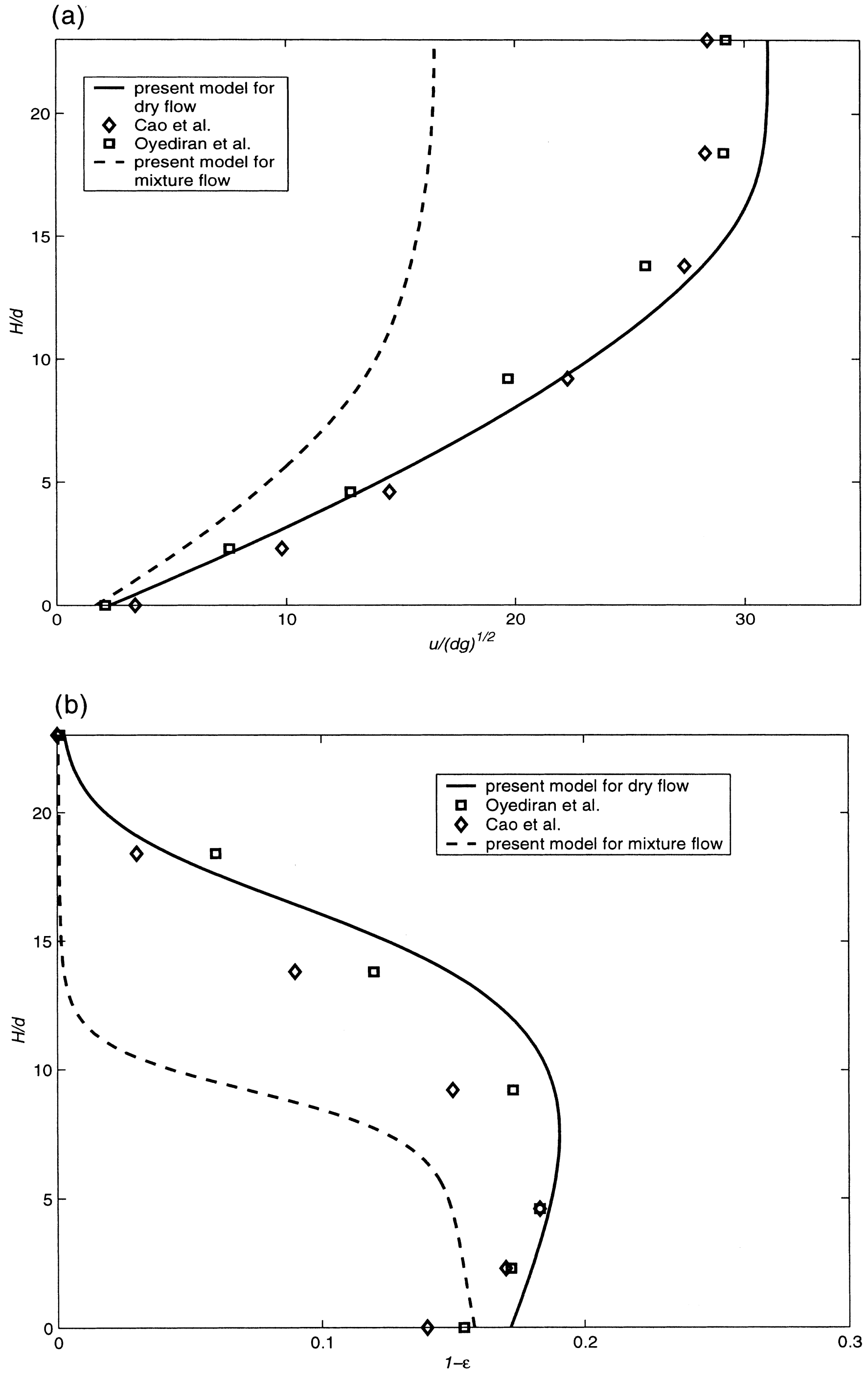


Fig. 1. $H/d = 23$, $d = 1$ mm and $v = 0$ for the granular–air mixture flow. Comparison with Oyediran et al. (1994) and Cao et al. (1996). Other parameters as in Table 1. Variation of (a) non-dimensional velocity, (b) particle volume fraction and (c) non-dimensional granular temperature with non-dimensional flow height.

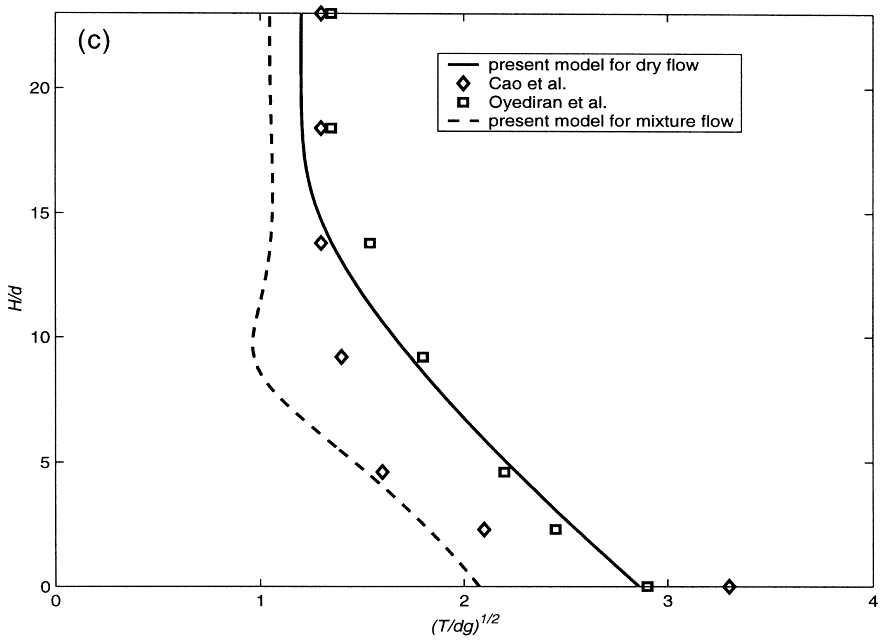


Fig. 1 (*continued*)

can be easily affected by the gas flow. The influence of increasing particle size is small for the larger particles. We will mainly consider relatively massive particles in the flow, because otherwise the air will play a more important role on the movement of smaller particles and our model approximations may become inappropriate. Although the profiles of velocity, temperature and volume fraction are similar for mixture flows with large particles, these flow patterns are still very different to the corresponding dry flows. In rapid granular–gas mixture flow at a fully-developed steady state, where the particle phase is dominating the flow, our model suggests that neglecting the influence of the interstitial gas will lead to significant inaccuracies, particularly because the influence of the interstitial gas 'accumulates' until the flow becomes steady.

Experimentally, it is very difficult to measure the real flow velocity of the interstitial air. Moreover, most researchers assume that the interstitial air can be neglected, so very little data is available. Drake (1991) reported that, in a chute flow where the interstitial air was at rest initially, the velocity of air was estimated as roughly half of the mean flow velocity of the particles at the fully-developed flow state. In the present model, the larger the relative motion between air and particle, the more damping influence the interstitial air has on the particle phase. When we assume the interstitial air is at rest, the final non-dimensional mean bulk velocity is the smallest. This can be seen in Fig. 3(a). Fig. 3 also shows that, with the relative velocity decreasing, the flow becomes similar to dry flow. Finally, when the interstitial air shares the same mean velocity with the particle phase, we obtain the same results as in dry flow. Physically, this is not entirely believable because the uniform air flow moving at the same mean bulk velocity will still dampen the

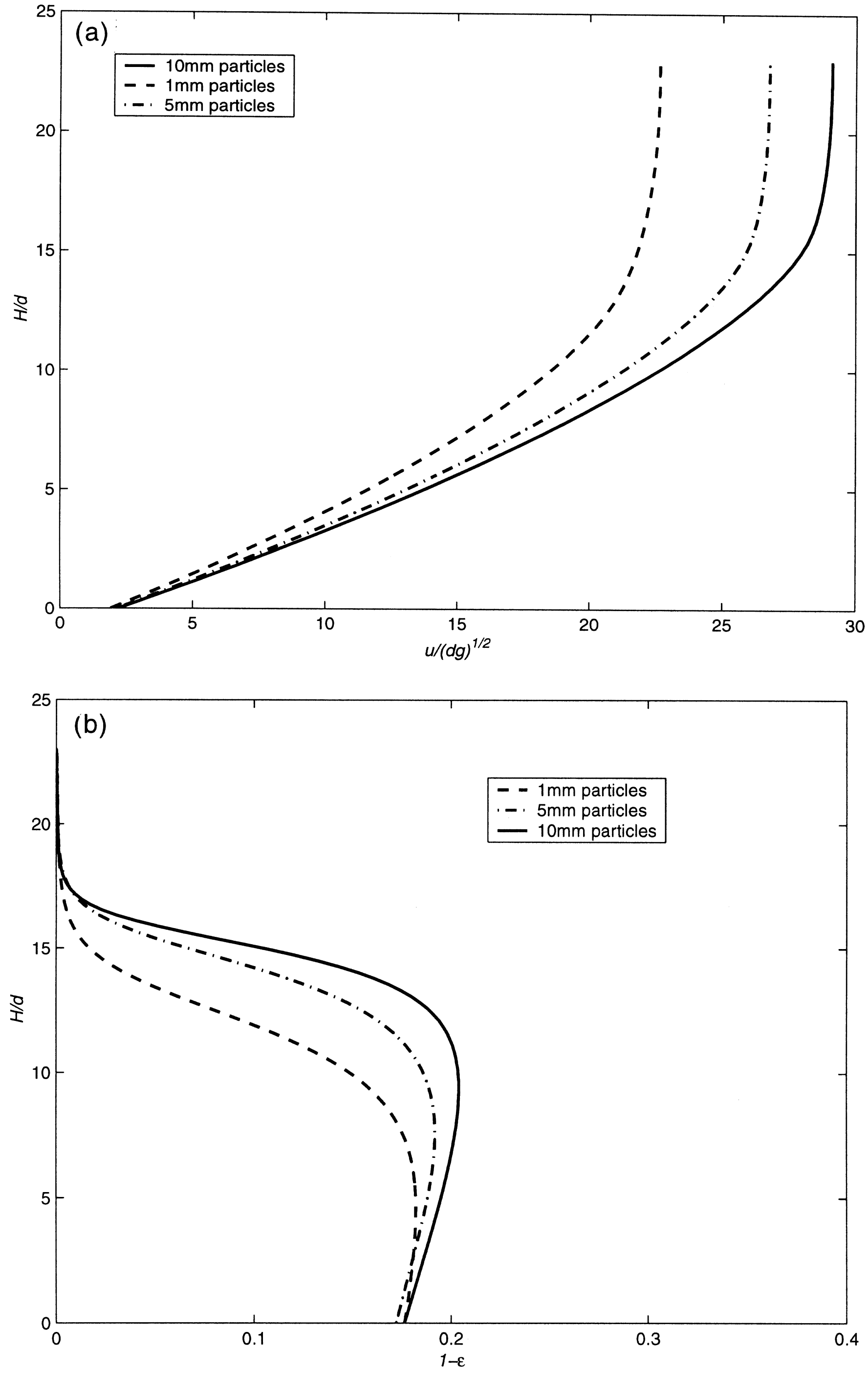


Fig. 2. $H/d = 23$, $d = 1$, 5 and 10 mm, respectively. Air velocity $v = 0.5u$ for the granular–air mixture flow. Variation of (a) non-dimensional velocity, (b) particle volume fraction and (c) non-dimensional granular temperature with non-dimensional flow height.

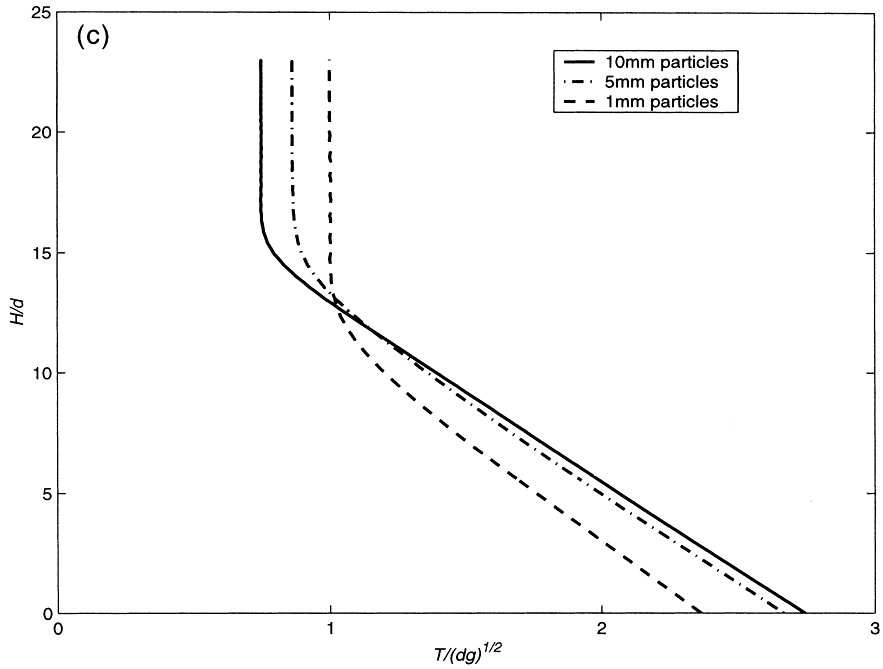


Fig. 2 (*continued*)

fluctuations of the individual particles in a volume element and will lead to a small energy dissipation. But, when the velocity of the interstitial air is not equal to the mean velocity of the bulk particles, the present model captures the main effect of energy dissipation due to drag in the air.

In Fig. 4(a) and (b), the profiles of shear and normal stress respectively are presented. In both flows, the stresses decrease from the wall boundary and tend to zero at the free surface as expected. It may be noted that the stresses decrease in nearly a linear way in the denser flow region. Compared to dry flow, the stresses in a mixture flow are smaller, as might be reasonably physically expected.

In Fig. 5(a), the energy lost due to inelastic collisions e_1 and due to the drag force e_2 are compared. In the same figure, the energy dissipations in both the dry and mixture flows are also compared. For a mixture flow, compared with inelastic dissipation, the drag dissipation is relatively small, especially in the region near the boundary wall. So it may be acceptable to neglect the effect of the drag force in establishing the boundary conditions at the wall. When the granular temperature decreases, the drag dissipation becomes more important until it is, indeed, greater than the inelastic collision dissipation. Both the dissipation mechanisms tend to zero at the free surface. Moreover, the inelastic dissipation is smaller in the mixture flow than in the dry flow. Fig. 5(b) also shows that the fluctuation energy flux is damped significantly by the interstitial air.

In Fig. 6, all the parameters are the same as in Fig. 1, except the flow height, which is now set to $H = 30$; changing the flow height will show the effect of altering the mass flow rate. We

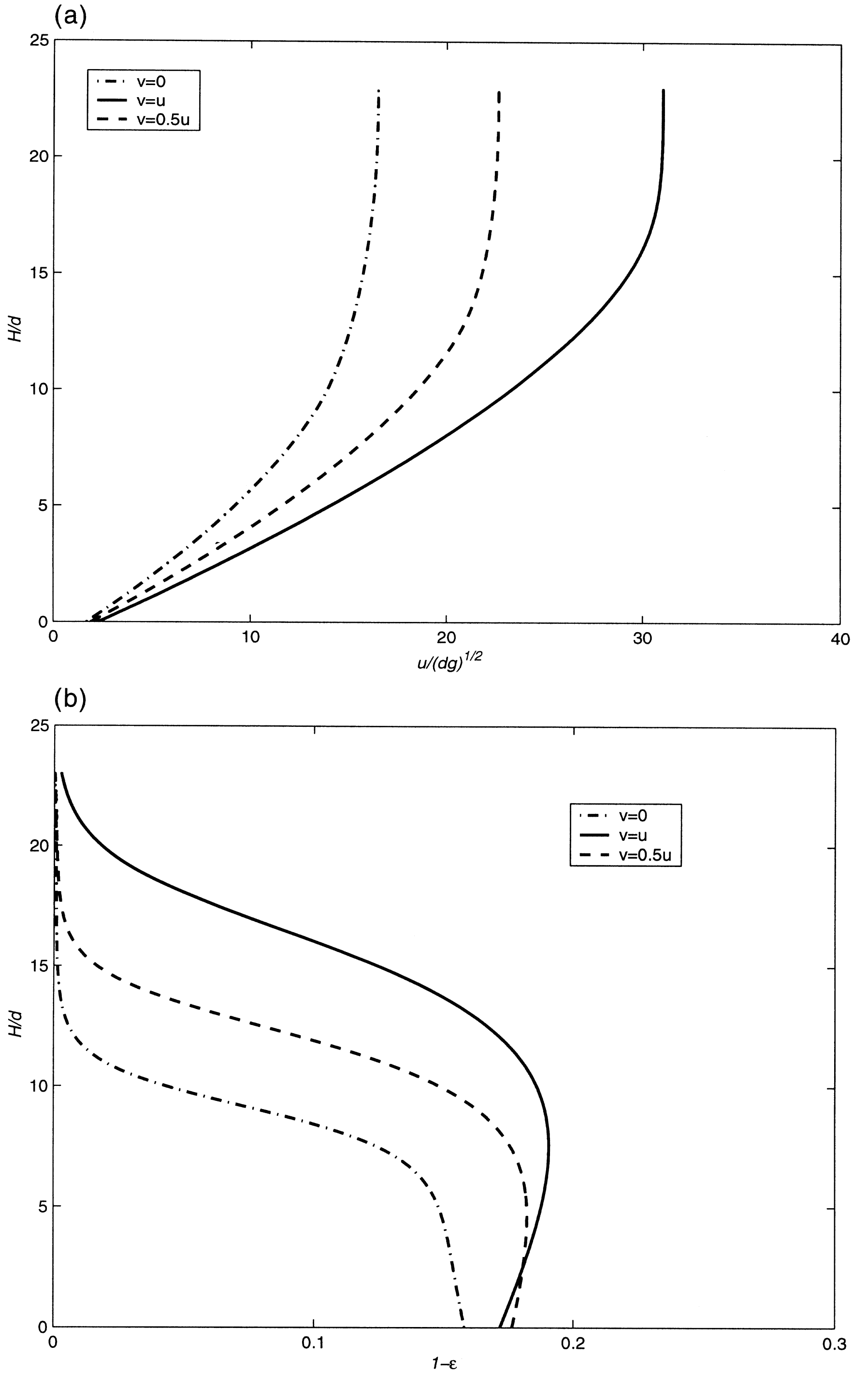


Fig. 3. $H/d = 23$, $d = 1$ mm, air velocity $v = 0$, $0.5u$ and u, respectively. Variation of (a) non-dimensional velocity, (b) particle volume fraction and (c) non-dimensional granular temperature with non-dimensional flow height.

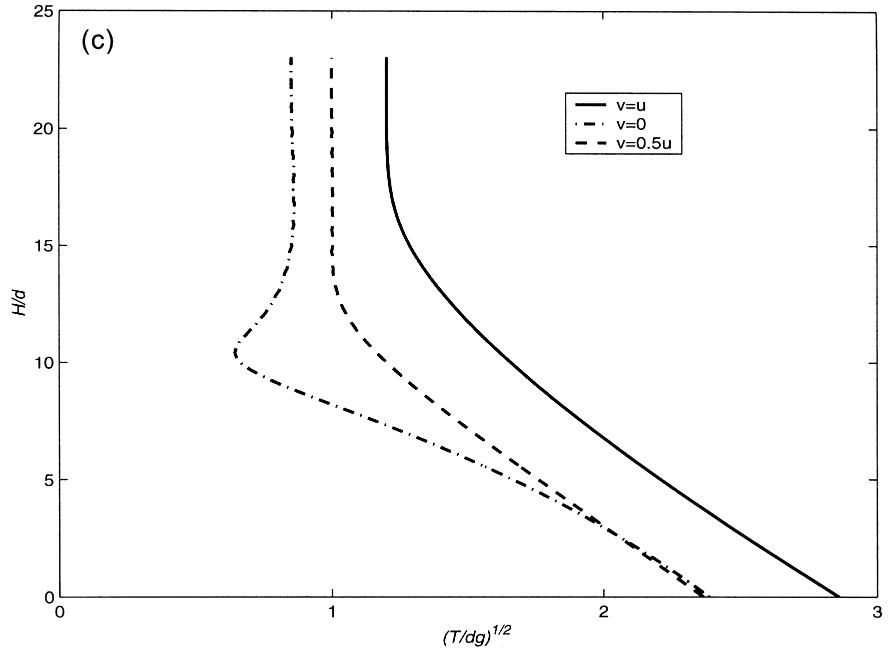


Fig. 3 (*continued*)

find that the interstitial air again plays a significant role in determining the flow profile. For larger particles, increasing particle size causes diminishing change in the results. A similar phenomenon can be observed here as in Fig. 2, i.e., the slip velocities of the different-particle-size flows and the dry flow are very close. This implies that the boundary geometry condition and the properties of the wall are the main factors in generating a slip velocity.

Sangani et al. (1996) proposed the following viscous energy dissipation rate, Γ ,for a fixed bed of particles:

$$\Gamma = 9\pi\mu_G dnTR_{diss}, \tag{45}$$

where n is the number density of the particles and R_{diss} is the so-called 'effective drag coefficient' of a spherical particle. Since $1-\varepsilon = (1/6)\pi nd^3$, our dimensionless form of this viscous energy dissipation rate is:

$$\gamma = 54(1-\varepsilon)R_{diss}\hat{\mu}_G\hat{T}, \tag{46}$$

where, $\hat{\mu}_G = \mu_G/(\rho_0 d^{3/2}g^{1/2})$, and R_{diss} is given by Sangani et al. (1996). A comparison of γ and w_1 (the non-dimensional form of our W_1 from Eq. (11)) is shown in Fig. 7. Here, w_1 may be caused by the interaction between fluctuations of the particles and the mean flow of gas, and γ may be interpreted as the energy which is dissipated by the interaction between the fluctuations

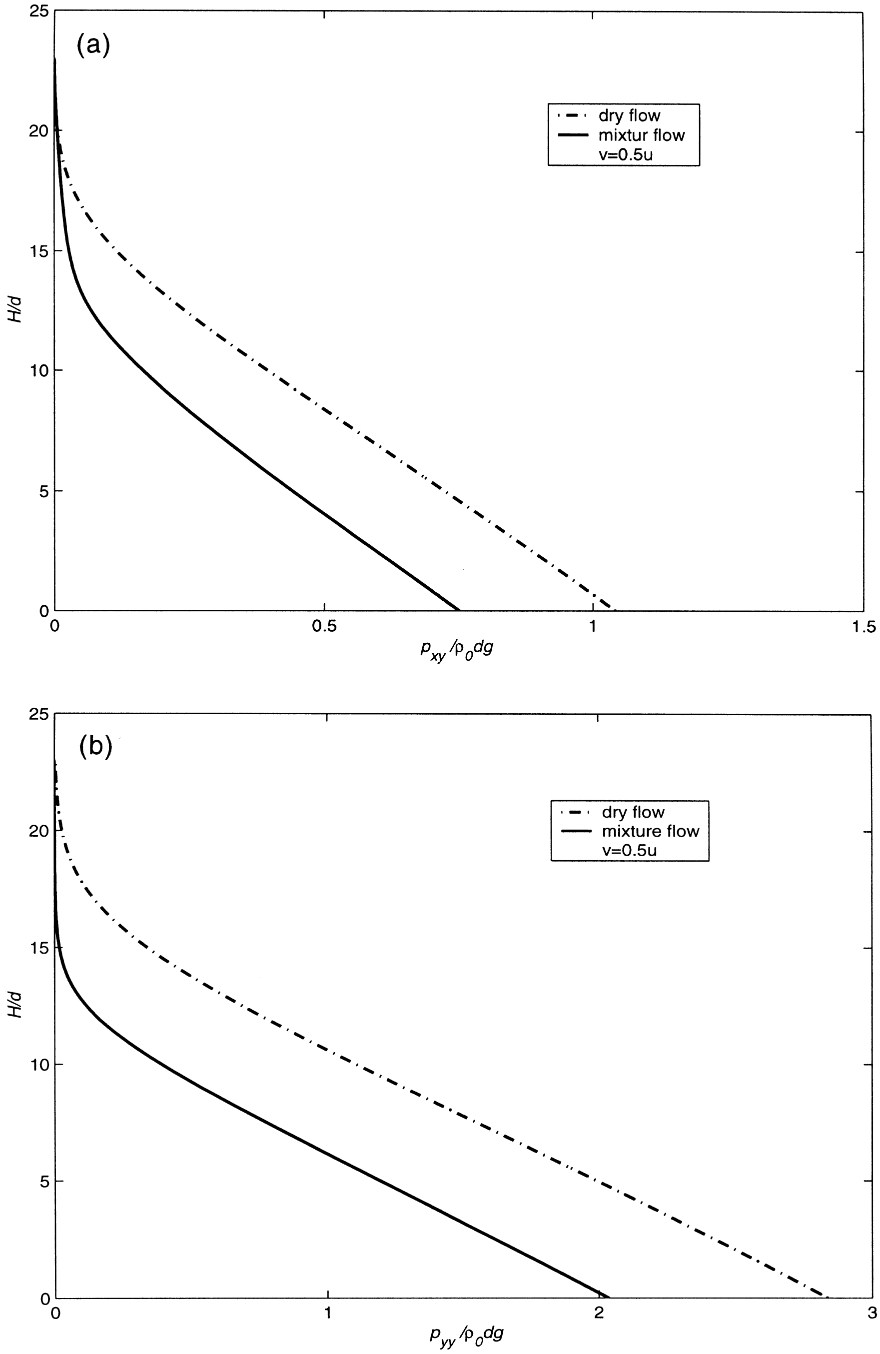
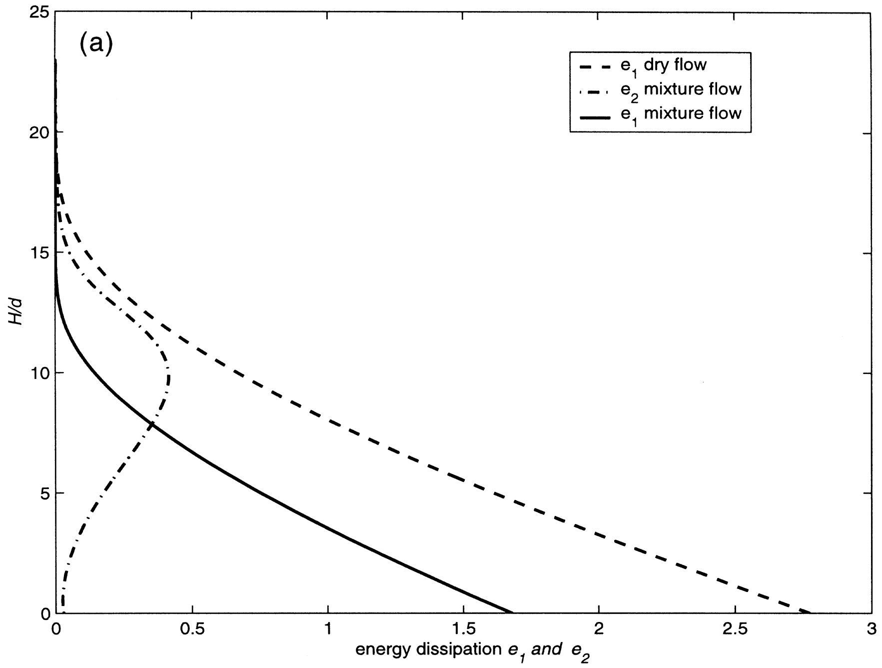


Fig. 4. $H/d = 23$, $d = 1$ mm, air velocity $v = 0.5u$. Variation of (a) non-dimensional shear and (b) non-dimensional normal stress with non-dimensional flow height.



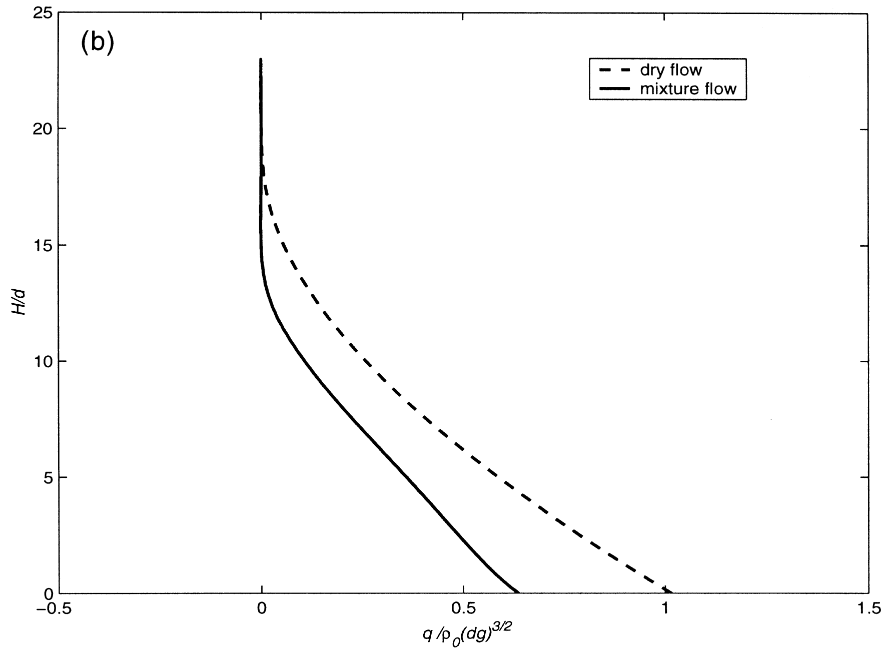


Fig. 5. $H/d = 23$, for granular–air mixture flow $d = 1$ mm, air velocity $v = 0.5u$: (a) non-dimensional energy dissipation by inelastic collisions in the dry flow and granular–air mixture flow, and non-dimensional energy dissipation by the interstitial air in the granular–air mixture flow; (b) the comparison of non-dimensional fluctuation energy in both flows.

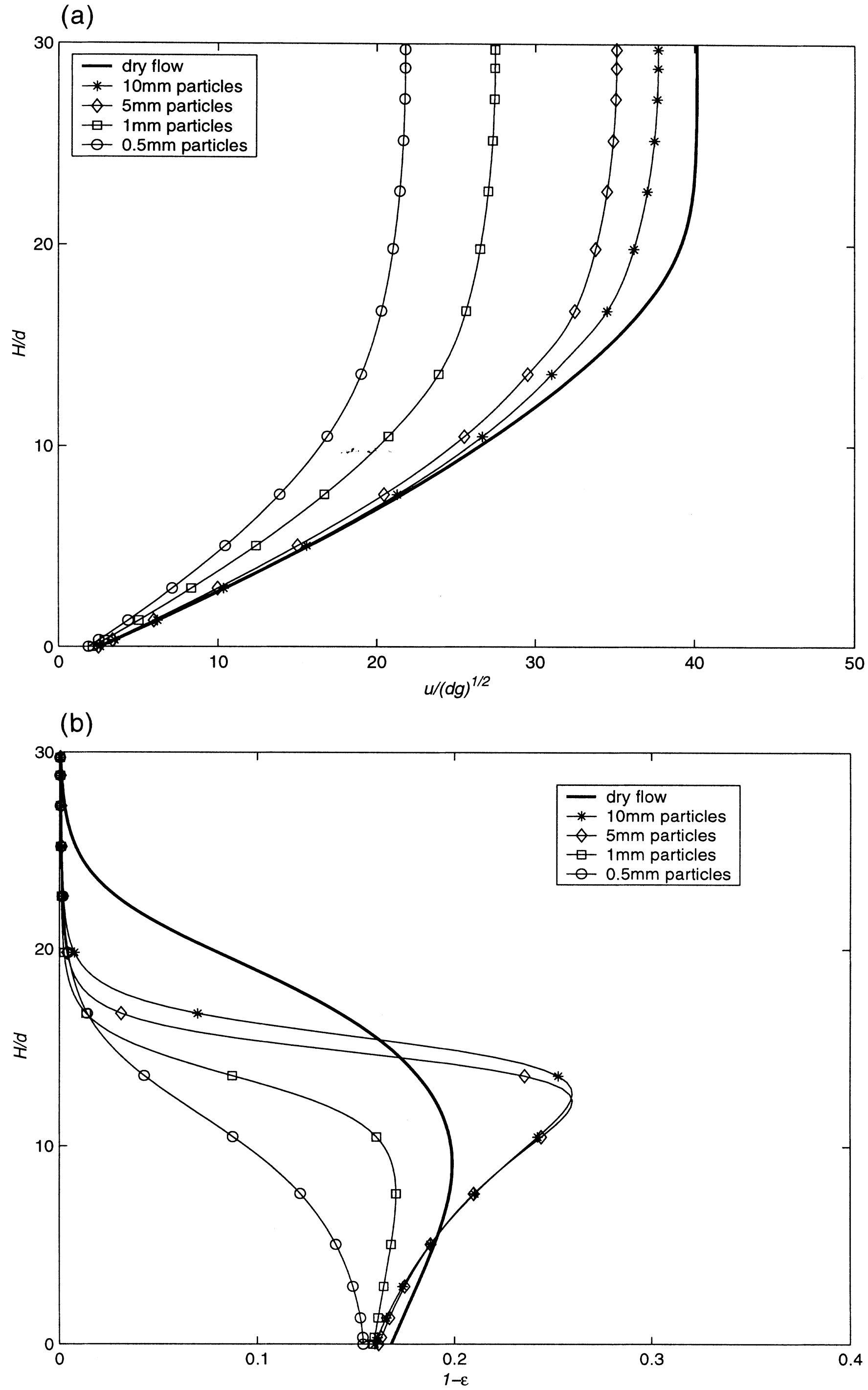


Fig. 6. $H/d = 30$, $d = 0.5$, 1, 5 and 10 mm, respectively. The interstitial air velocity $v = 0.5u$. Variation of (a) non-dimensional velocity, (b) particle volume fraction and (c) non-dimensional granular temperature with non-dimensional flow height.

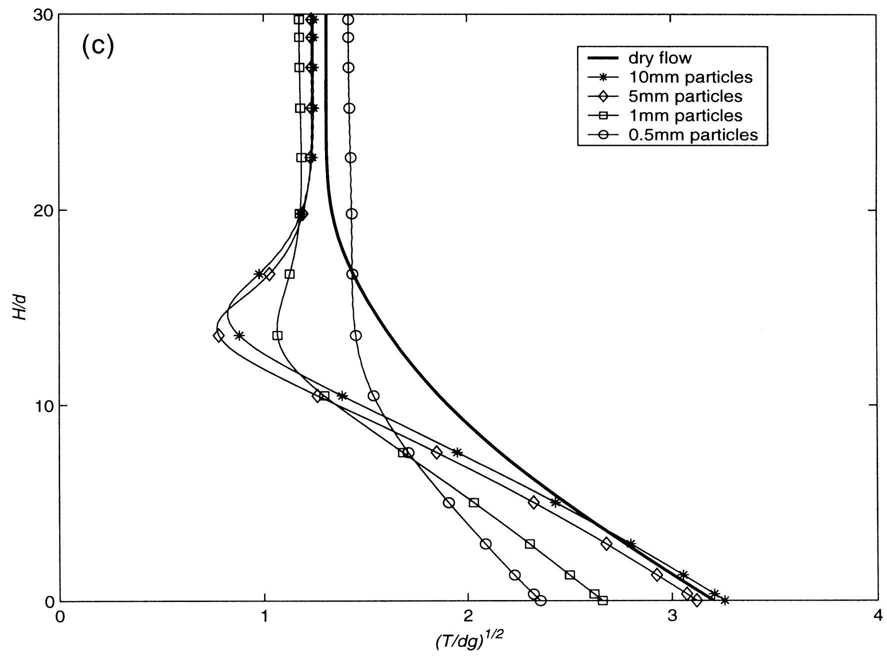


Fig. 6 (*continued*)

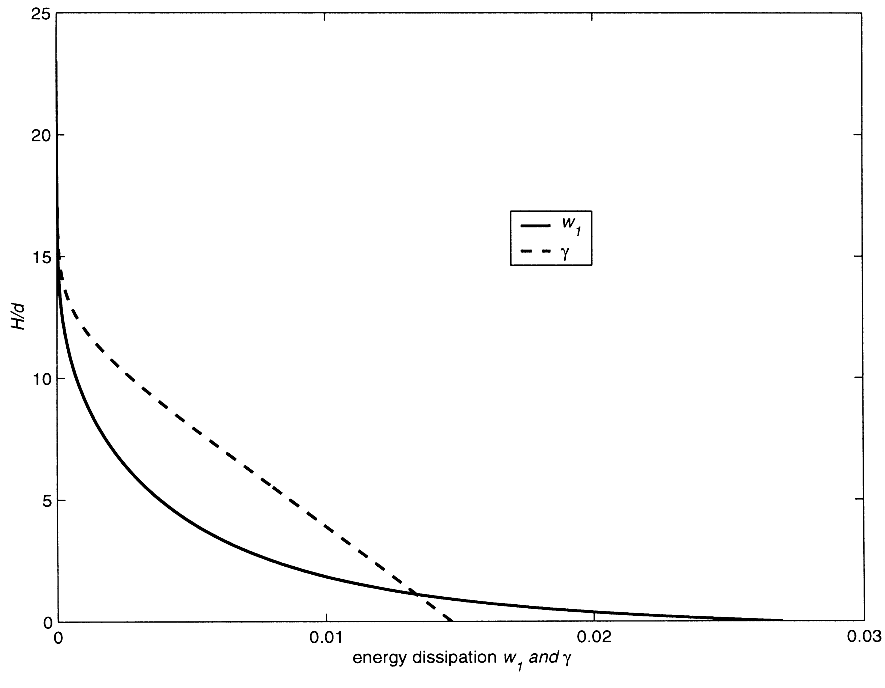


Fig. 7. $H/d = 23$, for a granular–air mixture flow with $d = 1$ mm, air velocity $v = 0.5u$. Comparison of two model non-dimensional energy dissipations, w_1 and γ.

in the two phases. Since the two terms are of the same order, if the expression of γ can be taken as valid for the rapid flow of particles down a chute, the fluctuation of the gas phase should be considered. We will, therefore, examine its influence in future work.

8. Conclusions

A new kinetic model which includes both the drag force and the energy dissipation due to an interstitial gas in the momentum equation and the energy equation, together with smooth bumpy boundary conditions at a wall, has been used to evaluate the steady-state profiles of granular velocity, solid volume fraction and granular temperature of granular–air mixture flows down an inclined chute under gravity. We may draw the following conclusions from the results:

- The interstitial gas plays an important role in a granular–gas mixture flow.
- Particle size can affect the flow: the flow of small particles is damped more by the interstitial gas.
- With larger particles, the effect on the flow of increasing particle size becomes smaller.
- There are large differences in the flow profiles at the fully-developed state between a mixture flow with large particles and a dry flow.
- The relative velocity between the particle phase and the gas phase greatly affects a mixture flow.
- Because of the large slip velocity at a smooth inelastic boundary wall, energy dissipation is largely via inelastic collisions between the particles and the wall. Hence, it is possible to neglect the influence of the interstitial gas in establishing the boundary conditions for momentum and energy balance in a thin flow layer above the wall.
- The non-dimensional slip velocity at the wall depends on the geometry of the wall and the coefficient of restitution between particles and wall not the interstitial gas, and the particle size.
- For a low granular temperature, the energy dissipated by the interstitial gas becomes more important than that dissipated by inelastic collisions.

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